

## **‘Izz al-Dīn al-Zanjānī and His Scientific Contribution to Mathematics and Astronomy**

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### **Abstract**

In this research paper, light is shed on unknown life of a forgotten mathematician. He achieved a fame through his writings and literary works that highlighted his scientific personality. He lived in a tense and dark period of Islamic history in Islamic East. This mathematician is ‘Izz al-Dīn al-Zanjānī, who lived in the first half of seventh century AH/ thirteenth century AD during Mongol invasion of Islamic East. We also try to enumerate his various works in mathematics and astronomy with their locations in different libraries of the world, and refer to books of his authorship that were attributed to others.

**Keywords:** Al-Zanjānī; Naṣīr al-Dīn al-Ṭūsī; ibn al-Fuwaṭī; Mongol invasion.

### **INTRODUCTION**

In this article, I will deal with the biography of ‘Izz al-Dīn al-Zanjānī, who has not received enough attention in studies, especially recent ones. Despite the multitude of biographers of him and cataloguers of his books, only a few information and lines can be drawn.

In referring the biography of ‘Izz al-Dīn al-Zanjānī, we will extract information regarding personalities we will discuss, as well as semantics of certain words and historical contexts, to gain a more comprehensive and clear understanding of al-Zanjānī’s life. By analyzing this data, we can choose the most suitable account in cases of dissimilarity, particularly when situated within its historical, temporal, and spatial context.

Sources agree that al-Zanjānī was distinguished in sciences of grammar, eloquence, conjugation, and prosody, in addition, they agree that he was alive in the seventh century AH / thirteenth century AD<sup>1</sup>. Some of these sources also mention that he is Shafī’i jurist (Brockelmann 1977: 179; Qara Bollut & Qara Bollut 2001 vol. 3: 1911). Through these sources and his writings, as we will mention in this article, we can discover places he traveled to, which are: Tabriz, Mosul, Khorasan, Bukhara and Baghdad.

Some biographical books of al-Zanjānī have caused confusion by mixing up ‘Izz al-Dīn ‘Abd al-Wahhāb al-Zanjānī with his father, ‘Imād al-Dīn Ibrāhīm al-Zanjānī. This is evidenced by certain works attributed to ‘Abd al-Wahhāb al-Zanjānī that were actually

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<sup>1</sup> Some of these sources are: (Ibn al-Fuwaṭī 1996 vol.1: 246-247; Ḥājī khalīyfah N.D. vol. 2: 1326; al-Ḥabashī 2004 vol. 1: 593; Kaḥāla, 1957, vol. 6, p. 216; Brockelmann 1977: 179; Madelung 2004: 841) ... And other sources, I did not mentioned in this article for their multiplicity.

authored by his father (Brockelmann 1977: 179). While some individuals believe that the son and the father are the same person (Ḥājī khaliyfah N.D. vol. 1: 412; vol.2: 1869). Sources, as we will see in the article, differ in his name, lineage, the date and the place of his death. They neglect another important aspect of his scientific personality; which is his mathematical personality; in which sources are silent. They did not indicate to those affected in this field.

Therefore, who is ‘Izz al-Dīn al-Zanjānī? What is his exact name and lineage? What is al-Zanjānī’s itinerary and movements? Did the historical period in which he lived affect his scientific production and his travels? What are the date and the place of his death? What are his scientific interests? What are his contributions to mathematics and astronomy? Who were his probable teachers he influenced by in mathematics? By looking at some of his mathematical writings and comparing them with his contemporary mathematicians, what does his mathematical level look like?.

### 1. Life of ‘Izz al-Dīn al-Zanjānī

Before starting narrates the biography of al-Zanjānī, it should be noted that the source we will depend without neglecting other sources or historical contexts, is the book *Majma‘ al-Ādāb fī Mu‘jam al-Asmā ‘alā Mu‘jam al-Alqāb* (Complex of Literature in Dictionary of Names and Nicknames) written by ibn al-Fuwaṭī (642-723 AH./1244-1323 AD.). Ibn al-Fuwaṭī<sup>2</sup> and al-Zanjānī lived the same historical events; they lived around the same period and the same geographical area.

Ibn al-Fuwaṭī’s accounts of al-Zanjānī were not transmitted or reported from other people, it is his own narration. He mentioned biographies of al-Zanjānī, his father and his son, in addition he met his son at Maraghah observatory in 570 AH/ 1272 AD. (Ibn al-Fuwaṭī 1996 vol. 1: 246-247; vol. 2: 9-10, 148).

Ibn al-Fuwaṭī recounted the following: “ ‘Izz al-Dīn abū Moḥamed ‘Abd al-Wahhāb ibn Ibrāhīm ibn Moḥamed al-Kharajī al-Zanjānī, the virtuous writer. Sojourner of Tabriz, he was virtuous, scholarly, wise, well-versed in transmitted and intelligible sciences. He settled in Tabriz. He had resided in Mosul... He was a scholar in grammar, language, conjugation, and sciences of meanings and rhetoric. He had traveled to Khurasan, crossed the river to Bukhara, and then he returned to Tabriz. When mawlānā al-Sa‘id Naṣīr al-Dīn al-Ṭūsī entered Tabriz, he asked him to classify an element in the science of *al-hay‘ah*, so he compiled for him the book entitled *al-Tadhkirah*. Among ‘Izz al-Dīn’s works is the book *al-Tadhkirah al-Majdiya* and others; he died in 660” (Ibn al-Fuwaṭī 1996 vol.1: 246-247).

In his own handwriting, ‘Izz al-Dīn al-Zanjānī wrote his exact name in his book *al-Kāfi Sharh al-Hādī*<sup>3</sup>, reading: “ ‘Izz al-Dīn ‘Abd al-Wahhāb ibn ‘Imād al-Dīn Ibrāhīm ibn ‘Abd

<sup>2</sup> Ibn al-Fuwaṭī: “The brilliant scholar, artificer, modernizer, useful historian, the pride of people of Iraq, Kamāl al-Dīn abū al-Faḍl ‘Abd al-Razzāq ibn Aḥmad ibn Muḥammad ibn abī al-Ma‘ālī al-Shaybānī ibn al-Fuwaṭī... He was born in Baghdad. He was captured in the battle < when Mongols marched on Baghdad > when he was young < he was 14 years old at the time >. Then, he came back to his teacher Khawāja Naṣīr al-Ṭūsī in the year sixty six hundred. He learned sciences of ancestors and he was more skillful than others in literature, history and people’s days (biographies). He was also adept in poetry and prose, and dexterous in writing biographies of people. He heard a lot and was concerned about this therefore he wrote, collected and reported. .... He stored Maraghah observatory books for a few ten years, and then he acquired valuable books and obtained histories. After Maraghah, he lived in Baghdad and took over the library of al-Mustansiriya and remained there as a librarian until his death. In fact, there were not any treasure books in the country than of these two libraries< the libraries in Maraghah and al-Mustansiriya in Baghdad >...” (al-Dhahabī 1919 vol. 4: 1493).

Notice: Phrases between two doubles<...> are my disposal for clarification.

<sup>3</sup>*Al-Kāfi Sharh al-Hādī* is an explanation of al-Zanjānī on his book *al-Hādī*.

al-Wahhāb ibn abū al-Ma'ālī al-Khazrajī al-Zanjānī"<sup>4</sup> (Ibn 'Abd al-Kāfī 1913: II).  
'Izz al-Dīn is the nickname<sup>5</sup>. The name is 'Abd al-Wahhāb. al-Zanjānī was nicknamed also by abū Muḥammad (Ibn al-Fuwaṭī 1996 vol.1: 246-247) attribution to his son Muḥammad. He has other nicknames written by scribes or commentators of his books,

<sup>4</sup> The text is taken from a footnote cited in Ishaq ibn Benjamin Yehuda's introduction in his edition of the book *Sharh al-Maḥnun bihi 'alā ghayri 'Ahlīhi*, in which he mentions the biography of al-Zanjānī. The following is an excerpt from this introduction in which the footnote appears, reading: "The work *al-Maḥnun bihi 'alā ghayri 'Ahlīhi* that I edit is the flower collection of Arabic poetry, which had been made by 'Izz al-Dīn 'Abd al-Wahhāb ibn 'Imād al-Dīn Ibrāhīm ibn 'Abd al-Wahhāb ibn abū al-Ma'ālī al-Khazrajī al-Zanjānī. The commentary of the work is from the pen of 'Ubayd Allah ibn 'Abd al-Kāfī.... .. Suyūṭī mentions al- 'Izzī in *Bughyat al-Wu'at fī Ṭabaqāt al-Lughawīn wa al-Nuḥāt*; he says he saw *al-kafī's* autograph commentary on *al-Hādī* which was completed on 20 Dīl Hīga 654 H/ 5 January 1257. However Suyūṭī does not allude to *al-Maḥnun*. This manuscript is in Cairo at Sultanian library where we have just seen it. Dates as well as the writer are those indicated by Suyūṭī <the footnote is placed here>." (Ibn 'Abd al-Kāfī 1913: I-II).

The footnote which cited by Ishaq ibn Benjamin Yehuda is the following: "here is the text of the end of this manuscript  
تم الكتاب والحمد لله على نواله، والصلاة والسلام على سيد الخلق محمد وآله، على يدي مصنفه عبد الله الفقير إليه أبي المعالي عبد الوهاب بن إبراهيم بن عبد الوهاب بن أبي المعالي الخزرجي الزنجاني في العشرين من ذي الحجة حجة أربع وخمسين وستمائة بمحروسة بغداد حرسها الله تعالى مع سائر بلاد المسلمين آمين يا رب العالمين. This manuscript that we went through ourselves is a large volume in 40, of a very beautiful handwriting, it measures 26 by 19 cms, and bears the number 66\* of the catalog of Sultannian library in Cairo, vol. IV. p. 88. It belonged to Ibrahim Pasha, the son of Mohammad Aly, then by inheritance to Prince Moustafa Pasha FadiI". (Ibn 'Abd al-Kāfī 1913: II).

Translation of arabic text cited above into English is: "The book was completed, and praise be to God for his blessing, blessings and peace be upon the master of creation, Muḥammad and his family, at the hands of his compiler, the poor servant of Allāh, abī al-Ma'ālī 'Abd al-Wahhāb ibn Ibrāhīm ibn 'Abd al-Wahhāb ibn abū al-Ma'ālī al-Khazrajī al-Zanjānī, on the twentieth of Dhul-Hijjah, Hajj of the year fifty-four and six hundred in the guarded city of Baghdad, may God guard it with the rest of Muslim countries, Amen, Lord of the world".

Al-Suyūṭī speaks of the manuscript mentioned by Ishaq ibn Benjamin Yehuda, and recounted that Zanjani's handwriting is of very good quality (al- Suyūṭī 1979: 122).

The introduction of Ishaq ibn Benjamin Yehuda was in French in the book *Sharh al-Maḥnun bihi 'alā ghayri 'Ahlīhi*, and it appears only in the copy directed to foreign countries, and includes a biography of 'Izz al-Dīn al-Zanjānī at the page II. While that *Sharh al-Maḥnun bihi 'alā ghayri 'Ahlīhi* is an explanation of the book *al-Maḥnun bihi 'alā ghayri 'Ahlīhi* which is attributed to 'Izz al-Dīn al-Zanjānī, explained by 'Ubayd Allah ibn 'Abd al-Kāfī. (Ibn 'Abd al-Kāfī 1913: II)

- Some sources mention the nickname of al-Harāmī (The thief) in the place of al-Khazrajī (for example at the beginning of the manuscript of 'Umdat al-Hussāb (Ahmet III, Topkapi A. 3457 ff 35 v. -101)). Karl Brockelmann referred to this nickname in his book History of Arabic Literature and said that it is a distortion of the nickname al-Khazrajī (Brockelmann 1977: 184).

- Sources also mention to the nickname al-Kharajī in place of al-Khazrajī. It seems that was cited by all those quoted ibn al-Fuwaṭī (such as Kaḥāla 1957 vol. 6: 216), (The encyclopedia of Islam (New Edition) 2004: 841 and Brockelmann 1977: 179). Ibn al-Fuwaṭī repeated this nickname also at al-Zanjānī's father and son nicknames, and it appears to be a distortion of Khazrajī. This happened, perhaps because ibn al-Fuwaṭī went young (14 old) to Maraghah with al-Ṭūsī, and as ibn al-Fuwaṭī's biographer said that he had learned Persian (Ibn al-Fuwaṭī 1996: 23), (al-Dhahabī 1919 vol. 4: 1493). For that, it is possible that the accent of ibn al-Fuwaṭī changed and he pronounced al- Kharajī instead al- Khazrajī.

<sup>5</sup> Kaḥāla mentioned in addition to nickname al- Zanjānī, the nickname al-Jurjānī, and he is the only one who mentioned to it (Kaḥāla 1957 vol. 6: 216).

-The nickname 'Izz al-Dīn is found in most biographers of al- Zanjānī and at the beginning of manuscripts attributed to him, among them:

- Ibn al-Fuwaṭī mentioned that the grandfather is Muḥammad, he did not mention the nickname of abū al-Ma'ālī for 'Izz al-Dīn al- Zanjānī, and the nickname of 'Imād al-Dīn was mentioned in the biography of Father Ibrāhīm al- Zanjānī (Ibn al-Fuwaṭī 1996 vol. 1: 246-247).

- Kaḥāla mentioned that the grandfather is Muḥammad and it seems that it was transmitted from Ibn al-Fawti, and he did not mention to abū al-Ma'ālī (the grandfather), he did not mention the nickname of 'Imād al-Dīn (Kaḥāla 1957 vol. 6: 216).

- Brockelmann attributed father's workbook to the son 'Izz al-Dīn al- Zanjānī, he did not mention the nickname abū al-Ma'ālī, and he did not mention the nickname of 'Imād al-Dīn (Brockelmann 1977:179).

- al-Zarkalī did not mention the nickname abū al-Ma'ālī for al- Zanjānī, or abū al-Ma'ālī for his grandfather (al-Zarkalī 2002: 179).

among them: Tāj al-Dīn<sup>6</sup> (al-Bābānī 1951:638), ‘Izz al-Batūl (Rosenfeld & Ihsanoğlu 2003: 207), abū al-Faḍā’il (Brockelmann 1977: 179).

The date of the birth of ‘Izz al-Dīn al-Zanjānī is unknown, and not much is known about his life. The place of his birth is probably Zanjan<sup>7</sup>, the city to which he belongs. He may have taken his first education by his father, so his father ‘Imād al-Dīn Ibrāhīm was virtuous, writer, scholar, jurist (Ḥājī khalīfah N.D. vol. 2: 9-10). ‘Imād al-Dīn Ibrāhīm had a short explanation of *al-Wajīz* from the book of *al-Rāfi’i* (d. 3-624 AH/6-1227 AD)<sup>8</sup>, he titled it *Naqāwat al-‘Azīz* (al-Subkī 1964: 119).

Therefore, in view of interests and contributions of each one of father and son, we can remove confusion between them, ‘Imād al-Dīn Ibrāhīm was a Shafi’i jurist, while ‘Izz al-Dīn al-Zanjānī was Adib and mathematician.

al-Zanjānī left his homeland Zanjan to Mosul (Ibn al-Fuwaṭī 1996 vol. 1: 246-247), perhaps in the year 617 AH./1220 AD., it is the year in which Genghis Khan began his first bloody campaign to conquer Muslim lands. Tatars killed many people, looted money and destroyed villages. They devastated in 617 AH./1220 AD. Transoxiana<sup>9</sup>, Khwarezm<sup>10</sup>, Khorasan<sup>11</sup>, Merv<sup>12</sup> and reached Azerbaijan capital of Tabriz at that time (Ibn al-Athīr 1987 vol. 10: 399-414).

When Mongol’s news reached these countries, whoever could escape fled to save his life. Probably, this also has done by al-Zanjānī. He left Zanjan to Mosul which was more secure, in which he was able to study, learn and work.

Many scholars had left their cities that were destroyed by Mongols to Mosul or other safe cities. For example, Yaqūt al-Hamawī (574 -626 AH./1178-1229 AD.), the author of the famous book *Mu’jam al-Buldān*, settled in Khwarezm before Mongol invasion, and when he was appalled by Mongols’s destruction, he quickly fled to Mosul (Ibn Khallikān 1977 vol. 6: 137).

Since the end of the 12<sup>th</sup> century and the beginning of the 13<sup>th</sup> century, Mosul had witnessed a boom in sciences. Rulers of Atābek state<sup>13</sup> established numerous educational institutions and schools. They approached scholars and writers, and gave them graces. Many eminent scientist and sheikhs came from different places either to learn from most famous scholars, or to teach and spread their knowledge (al-‘Abayjī 2009:56-57).

<sup>6</sup> Ḥājī was confused with Tāj al-Dīn ‘Abd al-Wahhāb ibn Ibrāhīm ibn ‘Abd al-Wahhāb al-‘Ardī al-Shafi’ī, the writer of *Sharh Marāh al-Arwāh*. While that *Marāh al-Arwāh* is an unknown writer who wrote it after the year 700 AH/1300 AD (Ḥājī khalīfah N.D. vol. 2: 1326; al-Ḥabashī 2004 vol. 3: 1653).

<sup>7</sup> Zanjan is an Iranian city located in the northwest of the country.

<sup>8</sup> *al-Wajīz* is a book on jurisprudence, while al-Rafi’i is a Shafi’i jurist.

<sup>9</sup> This river is the Amu Darya/ Oxus River, which is the boundary between Khorasan and the country beyond the river (the country beyond the river was known in the past as the country of Greater Turkestan, and when Arab Muslims conquered it in the first century AH, they called it: “The country beyond the river”, meaning the country located behind the river Oxus (Amordria), which historically formed the boundary between Persian and Turkish-speaking peoples, it is a vast area of great breadth, extending from Turkey in the west to the borders of China in the east, and has become divided into East Turkestan and West Turkestan, and the region of East Turkestan (Sinqiang), and what lies west are subject to this river is historically Khwarazm and Khorasan.

<sup>10</sup> Khwarazm is a large oasis located on delta of Oxus River in western Central Asia

<sup>11</sup> Historically: Khorasan includes "Islamic Khorasan" province of northwestern Afghanistan (such as the city of Herat) and parts of southern Turkmenistan, in addition to the present-day Khorasan province in Iran. Among its historical cities: Herat, Nishapur, Tus (known as Mashhad today), Balkh and Merv.

<sup>12</sup> Merv, or as it was known in the past as Merv al-Kubra or Merv al-Shahjan to distinguish it from Merv al-Rodh, is now the capital of the Mari region in Turkmenistan.

<sup>13</sup> The Atābek State is a ruling Islamic state that emerged in the western half of the Abbasid Caliphate. it was founded by ‘Imād al- Dīn Zankī in Mosul in the year 521 AH /1127 AD, and included during the reign of his son Nūr al- Dīn Mahmūd Mosul, al-Jazeera, al-Shām, Egypt and Yemen.

In the light of this prosperity, ‘Izz al-Dīn al-Zanjānī resided in Mosul (Ibn al-Fuwaṭī 1996 vol. 1: 247), seeking knowledge and then writing. al-Zanjānī’s linguistic proficiency was outstanding, and he authored several widely disseminated books on the subject, particularly in the subsequent century. Among his teachers in linguistics was Shams al-Dīn abū ‘Abd Ullāh, who was known as ibn al-Khabbāz (Ibn al-Fuwaṭī 1996 vol. 1: 247), nicknamed by abū al-‘Abbās. al-Zanjānī mentioned in his book *Sharh al-Qustās* that he recited it to his grammarian sheikh abū al-‘Abbās (Ḥājī khaliyfah N.D. vol. 2: 315). Ibn al-Khabbāz died in Mosul in 637 AH./1239 AD. (Fajjāl 1996:34).

The date of ibn al-Khabbāz’s death indicates that the period during which al-Zanjānī was a young man and a seeker of knowledge, was in the second or the third decade of the 7th century AH/13th century AD. Later he started writing, in Safar 637 AH./ September 1239 AD., al-Zanjānī was still staying in Mosul, in which he completed his book *al-Mu‘arrab ‘ammā fī al-Ṣīhāh wa al-Maghrib* at al-Qāhiriyya school (Ḥājī khaliyfah N.D. vol. 2: 1738).

Al-Zanjānī was also interested in mathematics in his youth. He had devoted a part of his life to this science, that’s what he said in his book *Qustās al-mu‘ādalāh fī ‘ilm al-jabr wa-al-muqābalah* (al-Zanjānī Ms; P.1); Yadegari 1980: 403). Unfortunately, neither sources nor he mentioned his teachers in mathematics.

But looking at that period in Mosul and in the presence of elite of encyclopedic scholars, it is possible to guess al-Zanjānī’s teachers in mathematics, and the most famous of them was Kamāl al-Dīn Mūsā ibn Yūnis (515-639 AH./1121-1241 AD.). Ibn Yūnis was in charge of al-Qāhiriyya School<sup>14</sup> (al-Zanjānī was one of its students) since its inauguration until 620 AH/1223 AD (Ibn Khallikān 1977 vol. 5: 316).

Ibn Yūnis was from a well-known scholarly family in Mosul. He taught for a very long time in its schools (al-‘Abayjī 2009: 54). He was well-versed in every part of sciences: logic, medicine, and mathematics such as mathematics of Euclid, *al-hay’ah*, conics, and *Almagest*. He knew types of calculus and arithmetic, algebra, double false position, music, *al-misāḥa* (Ibn Khallikān 1977 vol. 5: 312). In most of these sciences, al-Zanjānī wrote at least a book, as we will mention later in his works.

Ibn Yūnis was also the teacher of Naṣīr al-Dīn al-Ṭūsī (597-672 AH/1201-1273 AD)<sup>15</sup>. In his youth, al-Ṭūsī traveled from his city Tus (present-day Mashhad in Iran) to Nishapur in Khurasan, he was there before it was invaded by Mongols in 617 AH (Ridhawī 1998: 11-14). Until 628 AH, cities of Khorasan remained in ruins and no Muslims would dare to reside there (Ibn al-Athīr 1987 vol. 10: 419, 490).

From Nishapur, Naṣīr al-Dīn al-Ṭūsī fled to Merv, then he went to Baghdad and next Mosul (al-Amīn 1997: 16). This was in the period between 617 AH/1220 AD and the period 624 AH/1227 AD up to year 630 AH/ 1233 AD, during which he went to Ismaili castles where he remained there until its fall in 654 AH/1256 AD (Ridhawī 1998: 15, 20).

Ibn al-Fuwaṭī recounted that al-Ṭūsī wrote his book *al-Tadhkirah* in response to Zanjani's request (Ibn al-Fuwaṭī 1996 vol. 1: 247). al-Ṭūsī completed *al-Tadhkirah* in Maraghah in 659 AH./1261 AD., and in that time he was vizier of Hulegu (Ridhawī 1998: 43,289). At the beginning of *al-Tadhkirah*, al-Ṭūsī said; “We want to bring sentences from

<sup>14</sup> al-Qāhiriyya School was established by al-Qāher ‘Izz al-Dīn, who ruled Mouṣil (608-615 AH./1211-1221 AD.).

<sup>15</sup> Naṣīr al-Dīn al-Ṭūsī: He is Naṣīr al-Dīn Muḥammad ibn al-Ḥasan al-Ṭūsī, known as Khawāja Naṣīr, philosopher, scholar in mathematics and astronomy. He was accorded high status and he was obeyed by Hulegu. He was the disposer of money. He built Maraghah Observatory. (al-Dhahabī 2013 : 252).

the science of *al-hay'ah* as a memory to some of the loved ones..." (al-Ṭūsī after 1256: 1 v.). Although he did not mention him by name, he described al-Zanjānī as beloved one, it seems that their friendship is old, perhaps before they meet in Tabriz, al-Ṭūsī would not write to anyone, especially in that time in which he was in high position.

The period of al-Zanjānī's residency in Mosul seems to coincide with that of al-Ṭūsī, and it seems that the meeting between them happened for the first time when they were students at that phase of life of each one, that is why later in Tabriz, al-Ṭūsī wrote *al-Tadhkirah* at the request of his friend al-Zanjānī.

al-Zanjānī left Mosul to Tabriz<sup>16</sup> (Ibn al-Fuwaṭī 1996 vol. 11: 246- 247)<sup>17</sup>. Tabriz was not affected by the destruction befell Transoxiana and Khorasan. Tatars reached it firstly in 617 AH/1220 AD, next in 618 AH/1221 AD, then in 620 AH/ 1223 AD, after that in the second campaign of Tatars in 628 AH/1231 AD. They did not loot Tabriz, because in every time, Tatar rulers reconciled with Tabriz rulers over money (Ibn al-Athīr 1987 vol. 10: 409, 415, 472, 495).

Tabriz had been handed over to Mongols without resistance at the beginning of the year 629 AH/ 1232 AD (al-Ṣallābī 2009: 174). Unlike Genghis Khan Leader of the first campaign of Tatars, his succeeds established countries they owned, one of them is Menkukhan (grandson of Genghis Khan and brother of Hulegu), who began his reign in the year 648 AH/ January 1250 AD.

Mankukhan ordered his brother Hulegu, when he led him in the army in the third campaign, not to aggress people under their rule except those who resist; they repopulated countries under their rule (al-Ṣallābī 2009: 190).

al-Zanjānī sojourned in Tabriz in the first time he go to it for a short time, Where he completed his algebraic book *Qusṭas al-mu'ādalāh fī 'ilm al-jabr wa-al-muqābalah* in Thursday 28 Muharram 643 AH/ 01 July 1245 AD (al-Zanjānī Ms<sub>b</sub>: 226 r.).

Later al-Zanjānī left it to Khorasan, and then he crossed the river (Gihun River) to Bukhara (Ibn al-Fuwaṭī 1996 vol. 1: 246-247). In it, al-Zanjānī completed his book *Me'yār al-Nuẓẓār fī 'Ulūm al-Ash'ār*, on Thursday, 21 Jumada al-Thani 650 AH,/ 4<sup>th</sup> August 1252 AD. (al-Zanjānī 1991: 30).

At that time, it is mentioned that Menkukhan's mother, who was Christian, had behaved well with Muslims, especially imams and sheikhs. She set up a school in Bukhara at her own expense and made many endowments for it, and she entrusted it to Sayf al-Dīn al-Bakhrizī. She appointed teachers and gave alms to poor, and this was her habit until she died in the year 649 AH/1251 AD (al-Ṣallābī 2009: 185-186).

In Sha'ban of 653 AH./1255 AD., Hulegu's army reached Samarkand, and from there they went to Khorasan. In Dhul-Hijjah of the same year, Hulegu issued orders to stop all ships and boats, and to build a bridge on Oxus River. His forces crossed the river towards Ismaili castles (al-Ṣallābī 2009: 190, 192), and thus began the third Tatar campaign. Probably, this news made al-Zanjānī leave Bukhara towards Baghdad.

In Baghdad, al-Zanjānī completed at least three books: *Sharh al-Hādī* in Dhul-Hijjah in 654 A.H/ January 1256 AD (al-Suyūfī 1979: 122), *al-Taṣrīf al-'Izzī* in 655 A.H/ 1257 AD (Brockelmann 1977:179), and his explanation of *Taṣhīh al-Qiyās fī Tafsīr al-Qusṭās* in 655 A.H/ 1287 AD (Ḥājī khalīyfah N.D. vol.1, p. 315), which is an explanation of al-Zamakhsharī's book (d. 538 A.H./ 1144 AD.) *al-Qusṭās* in prosody.

<sup>16</sup> Tabriz is one of the most important and prominent cities in Iran, it is the capital of East Azerbaijan Province.

<sup>17</sup> According to ibn al-Fuwaṭī, 'Izz al-Dīn al-Zanjānī traveled to Tabriz twice. In the first time he was sojourner, which means that he stayed for a short period (few days, months, or few years).

Several guesses have been given about the place and the date of al-Zanjānī's death and his life after the year 655 AH./1257 AD. Most of these guesses, either did not specify a date for his death and mentioned that he was alive in the year 654 AH/1256 AD (Ḥājī khalīyfah N.D. vol. 1: 412; vol. 2: 1869), or who said that he died in the year 655 AH/1257 in Baghdad (al-Zarkalī 2002: 179). There is who imposed the possibility of his death in Baghdad during Mongol invasion in the year 656 AH/1258 AD (Shahwī & Baqirī 2016: ٥٥), or who mentioned that he died in Baghdad in the year 660 AH/1262 AD. (kaḥālah 1957: 216).

Our most likely opinion is that: al-Zanjānī was lived after Mongol invasion of Baghdad and he left to Tabriz, for the following reasons:

1. According to ibn al-Fuwaṭī, 'Izz al-Dīn al-Zanjānī traveled to Tabriz twice, the first time he was sojourner, the second time he returned to settle it (Ibn al-Fuwaṭī 1996 vol. 1: 247), which indicates that he returned to it to spend the rest of his life<sup>18</sup>. So, when al-Zanjānī returned to Tabriz? It is conceivable that al-Zanjānī settled in Tabriz after all his travels.

2. When al-Ṭūsī entered Tabriz, al-Zanjānī asked him to write something in the science of *al-hay'ah*, al-Ṭūsī wrote *al-Tadhkirah* at the request of al-Zanjānī (Ibn al-Fuwaṭī 1996 vol. 1: 247). *al-Tadhkirah* also called *al-Tadhkirah al-Naṣīriyah*. Al-Ṭūsī had written three books in the science of *al-hay'ah* before *al-Tadhkirah al-Naṣīriyah*, which are *al-Risālah al-Mu'iniyyah fī 'ilm al-hay'ah*, *Sharḥ-i risālah-yi Mu'iniyyah* and *Zubdat al-hay'a* (in Persian). The book *al-Tadhkirah al-Naṣīriyah* had many explanations. Al-Ṭūsī completed it in Maraghah at 659 AH/1261 AD (Ridhawī 1998: 276, 279, 280, 289). The question is: when the meeting happens between al-Ṭūsī and al-Zanjānī in Tabriz?

- As we mentioned above, sources mentioned that al-Ṭūsī was in Tus from his childhood to his youth, then he went to Nishapur in Khorasan, he left it to Merv to escape from the first bloody campaign of Mongols in 617 AH./1220 AD., then to Baghdad, after that to Mosul (al-Amīn 1997: 16), then he went to Ismaili fortresses Alamūt where he stayed for a long time until its fall in 654 AH/1256 AD, then he was respected and he was obeyed by Hulegu who accompanied him during the march to Baghdad (Ridhawī 1998: 15, 20-21).

It is clear that al-Zanjānī did not meet al-Ṭūsī during his first stay in Tabriz (al-Zanjānī Ms<sub>B</sub>: 226 r.), because al-Ṭūsī was residing at that time in Alamūt. Therefore, we deduce of above that is impossible for al-Ṭūsī and al-Zanjānī to meet in Tabriz before the year 656 AH/ 1258 AD, the date of Mongol invasion of Baghdad. But sources mentioned that al-Ṭūsī and Hulegu were on borders of Tabriz on 4<sup>th</sup> Sha'ban 657 AH/ 2 August 1259 AD (Ridhawī 1998: 37; al-Hamadhānī 1960: 301-302). It is probable that he met al-Zanjānī at that time, when al-Zanjānī returned to it the second time when he settled there.

- If we assume that the meeting between al-Ṭūsī and al-Zanjānī took place before Mongol invasion to Baghdad (Assuming that sources did not mention that Tusi went to Tabriz before Mongol invasion), that mean, it will be Either between the fall of Ismaili castles in 654 AH/ 1256 AD and the fall of Baghdad in 656 AH./ 1258 AD., or before the entrance of Nasir al-Din al-Ṭusi to Ismaili castle.

In the case one, al-Zanjānī was in Baghdad as we mention above (al-Suyūṭī 1979:122; Brockelmann 1977: 179; Ḥājī khalīyfah N.D. vol.1: 315).

In the second case, the meet it will be at least before al-Ṭūsī entered Ismaili fortresses Alamūt, from which he did not leave until it fell into hands of Mongols, the date of his entry into the castle is unknown, but al-Ṭūsī began composing his first writings for Ismaili

<sup>18</sup> Ibn al-Fuwaṭī said in arabic language « استوطن تبريز », that mean he made Tabriz home (his contry).

castles' ruler in 632 AH/1235 AD (Ridhawī 1998: 15, 20). Therefore the meeting was at least before 632 AH/1235 AD. It is inconceivable that al-Ṭūsī would fulfill his friend's wish after more than 27 years of request.

- Al-Ṭūsī wrote *al-Tadhkirah al-Naṣīriyah* in 659 AH./1261 AD. and would compose it for a living person, not a dead person. It is not the custom of people of that time to write and dedicate to a dead person as in the present. Therefore al-Zanjānī was alive in 659 AH./1261 AD.

3. Ibn al-Fuwaṭī (642-723 AH./1244-1323 AD.) met the son of al-Zanjānī, and while he presents his biography, he recounted that: "He is 'Imād al-Dīn abū al-Makārim Muḥammad ibn 'Izz al-Dīn al-Zanjānī 'Abd al-Wahhāb ibn Ibrāhīm al-Kharajī al-Zanjānī, then al-Tabrīzī" (Ibn al-Fuwaṭī 1996 vol. 2: 148). This text indicates that, 'Imād al-Dīn Muḥammad son of al-Zanjānī was originally from Zanjān, and then he wills Tabrizian (Tabrīzī), that mean he settled in Tabrīz. This further reinforces the hypothesis that Izz al-Dīn al-Zanjani moved with his family to Tabriz after leaving Baghdad.

From what we mentioned above, it can be said that al-Zanjānī with his family left Baghdad to Tabriz. We do not know whether this happened before the invasion, or that al-Zanjānī somehow survived during the march of Mongols and left Baghdad, in that time and during reign of Hulegu, Tabriz became one of major metropolises with Maraghah in Ilkhanat state<sup>19</sup>.

Later, al-Zanjānī asked al-Ṭūsī to write something in *al-hay'ah*, according to his desire, al-Ṭūsī wrote *al-Tadhkirah al-Naṣīriyyah* (Ibn al-Fuwaṭī 1996 vol. 1: 247), Maybe the meeting between them happened when al-Ṭūsī was in Tabriz in 657 AH./1259 AD. (al-Hamadhānī 1960: 301-302).

During this period of his life in Tabriz; it seems that al-Zanjānī continued composes books, and *al-Tadhkirah al-Majdiyya* mentioned by Ibn al-Fuwaṭī was one of them. Despite fame of al-Zanjānī's other works; especially in literature; *al-Tadhkirah al-Majdiyya* is the only work mentioned by ibn al-Fuwaṭī (Ibn al-Fuwaṭī 1996 vol. 1: 247).

It seems that the name of the book was stuck in ibn al-Fuwaṭī's mind, because al-Zanjānī wrote it in the end of his life, which coincided with the beginning of ibn al-Fuwaṭī's life, and perhaps the name of this book is *al-Tadhkirah* like the well-known book of Naṣīr al-Dīn al-Ṭūsī, or maybe the reason is the person to whom al-Zanjānī dedicated the book.

It seems that al-Zanjānī wrote this book for someone of nobles, probably named Majd or Majd al-Dīn. If we look at coincidence of historical contexts with the place, it is most possible that the person is Majd al-Dīn abū al-Muẓaffar 'Abd al-Majīd ibn Muḥammad al-Tabrīzī.

Majd al-Dīn abū al-Muẓaffar al-Tabrīzī king of Tabriz is president of Azerbaijan. He is the chief and great king. He was closed and specialized in the presence of Hulegu. Ibn al-Fuwaṭī saw him in Tabriz in 657 AH./1259 AD. (Ibn al-Fuwaṭī 1996 vol. 4: 457), Noting that is the year in which al-Tusi was in Tabriz with Hulegu (Ridhawī 1998 : 35, 37).

Majd al-Dīn al-Tabrīzī is one of Hulegu's trustworthy friends. Hulegu sent him treasuries and money that he brought from Baghdad to Azerbaijan, and all what he looted from fortresses of Ismailia, Romans, Karaj, Armenians and Kurds and commissioned Majd al-Dīn al-Tabrīzī to build a high and narrow building to melt treasures in it (al-Hamadhānī 1960: 300).

In this context, we can point out that this conjecture is an additional argument that al-

<sup>19</sup> Mongol Ilkhanat era began to form during the reign of Genghis Khan (d. 1227 AD.), but It was not a state in itself except during the reign of Hulegu (654-663 AH./1256-1265 AC.).

Zanjānī was alive and met al-Ṭūsī in Tabriz perhaps in year 657 AH./1259 AD. It appears that *al-Tadhkirah al-Majdiyya* is in *al-hay'ah* due to the intense interest of Mongols in astronomy and astrology.

‘Imād al-Dīn Muḥammad son of ‘Izz al-Dīn al-Zanjānī was writer and poet. He was very intelligent and eloquent. He was in charge of the endowments of Tabriz and its works. He praised Shams al-Dīn Muḥammad ibn Muḥammad al-Juwaynī (Ibn al-Fuwaṭī 1996 vol. 2: 148).

‘Imād al-Dīn Muḥammad worked in endowments during Mongol era because the praise was for Shams al-Dīn al-Juwaynī owner of the divan (the Ministry of Finance). He had the authority to dispose money carried to him from all Tatar countries during the era of Mongol Ilkhanat (al-Nuwayrī 2004: 270).

Hulegu put all endowments belonging to metropolises at the disposal of Naṣīr al-Dīn al-Ṭūsī to invest tenths of them in paying expenses related to Maraghah observatory (Ridhawī 1998: 41), therefore, it is probably that Naṣīr al-Dīn al-Ṭūsī was the one who authorized son of ‘Izz al-Dīn al-Zanjānī to work in endowments.

Finally, in the light of what we have mentioned, we can say that al-Zanjānī and his family settled after Mongol invasion of Baghdad in Tabriz, and he died in it the year 660 AH/1262 AD (Ibn al-Fuwaṭī 1996 vol. 1: 247).

## **2. The Scientific Works of ‘Izz al-Dīn al-Zanjānī**

### **A. In Mathematics:**

#### **a. In Arithmetic and Calculus**

##### **1. ‘Umdat al-Ḥisāb:**

It was referred to as one of al-Zanjānī’s works by: (al-Zarkalī 2002: 179; Qara Bollut & Qara Bollut 2001 vol. 3: 1911; Rosenfeld & Ihsanoğlu 2003: 207).

Copies of the manuscript are in following libraries:

- Istanbul, Topkapı Sarayı Müzesi, Ahmet III, N° 3145.
- Istanbul, Topkapı Sarayı Müzesi, Ahmet III, N° 3457.

The book is on mental arithmetic, known at that time *ḥisāb hawā’ī*. The book contains 12 chapters. It includes three main topics. The first topic is about names of numbers, its types, ranks, properties, and operations on it (multiplication, division, ratio, rooting).

The second topic that the book deals with is theory of numbers, among topics studied is: properties of some numbers and their summations, as well as many Diophantine equations.

The third topic in which al-Zanjānī deals is: issues and their solutions, as he mentions three ways to solve these issues, which are: fourth proportional method, inverse method, and double false position method.

##### **2. Risāla Kāfiya fī ‘ilm al-Ḥisāb:**

It was referred to as one of al-Zanjānī's works by: (Qara Bollut & Qara Bollut 2001 vol. 3: 1911; Rosenfeld & Ihsanoğlu 2003: 207).

We have to notice that, Berlin Library Catalog at the end of description of treatise entitled *Risāla Kāfiya fī ‘ilm al-ḥisāb al hawā’ī* refers to Hājī Khalīfa (1017-1068 AH. / 1609-1657 AD.) (Ahlwardt 1893 vol. 5: 351), who attributed to ibn-as-Samḥ, Asbaḡ Ibn-Muḥammad two books *al-Kāfi fī ‘ilm al-ḥisāb al hawā’ī* and *al-Kāmil fī ‘ilm al-ḥisāb al hawā’ī*, although Hājī Khalīfa on the same page attributed to ‘Izz al-Dīn al-Zanjānī, *Kāfiya fī ‘ilm al-ḥisāb* (Hājī khalīyfah N.D. vol. 2: 1377), so in Berlin Library Catalog we found

that *Risāla Kāfiya fī ‘ilm al-ḥisāb al hawā’ī* (Ms. or. Quart.100, Staatsbibliothek (Berlin)) was attributed to Ibn-as-Samḥ, Asbaḡ Ibn-Muḥammad.

Escorial Library transmitted the reference of Berlin Library Catalog, and referred to Suter who mentioned to two books of ibn al-Samḥ, who in turn said that Hājī Khalīfa cited them among ibn Ibn-as-Samḥ’s books (Suter 1900:85), and since the beginning and topics are the same in two treatises of Escorial Library and Berlin Library, so it is remained and circulated as a treatise of Ibn-as-Samḥ not of al-Zanjānī.

Copies of the manuscript are in following libraries:

- Istanbul, Topkapı Sarayı Müzesi, Ahmet III, N° 3152, which is beginning by:  
”الحمد لله رب العالمين والصلاة والسلام على سيد الخلق محمد وآله الطيبين الطاهرين وبعد، فهذه رسالة كافية في علم الحساب رتبناها على عشرة فصول...”

It is ending by:

”... ولا يخفى عليه ما يرد من جنسها وليكن هذا آخر كلامنا في هذه الرسالة والحمد لله رب العالمين، تم.”

- Madrid, Escorial, N° 973, it is incomplete, and it is beginning by:  
”الحمد لله رب العالمين والصلاة والسلام على سيد الخلق محمد وآله الطيبين الطاهرين وبعد، فهذه رسالة كافية في علم الحساب رتبناها على عشرة فصول...”

It is ending by:

”... وهو ستة في الخط الأول فيكون سبعة وعشرين ونصف نقسمها على مجموع الخطأين فيخرج...”

- Berlin, Staatsbibliothek, Ms. or. Quart.100, entitled *Risāla Kāfiya fī ‘ilm al-ḥisāb al hawā’ī*; it is beginning by:  
”الحمد لله رب العالمين والصلاة ... وبعد فهذه رسالة كافية في علم الحساب الهوائي رتبناها على عشرة فصول...”

It is ending by:

”... ولا يخفى عليه ما يرد من جنسها وليكن هذا آخر كلامنا في هذه الرسالة تمت.”

- Mashhad, Iran, Endowment Hajj Mawla Nowruz ‘Alī al-Bastamī, entitled *Risāla Kāfiya fī al-ḥisāb*, (al-Ṭahrānī 1983: 249), it is beginning by:  
”الحمد لله رب العالمين والصلاة والسلام على سيد الخلق محمد وآله أجمعين ، أما بعد فهذه رسالة كافية في علم الحساب رتبناها على عشرة فصول ، الفصل الأول في معرفة الأسامي والمراتب”

- Damascus, al-Ṭahiriyyah Library, N° 6000, in a collection occupies pages 104-146, entitled *Risāla Kāfiya fī ‘ilm al-ḥisāb*, it was completed on Sunday, 27 Rabi’ al-Thani in the year 799 AH. (‘Āyḍī 1973:15-16).

It is beginning by:

”رب يسر، قال الشيخ الإمام العلامة...فهذه رسالة كافية في علم الحساب رتبناها على عشرة فصول، الفصل الأول في معرفة الأسامي”

It is ending by:

”... فإن المحصل إذا أتقن هذه المسائل لا يخفى عليه ما يرد من جنسها، وليكن هذا آخر كلامنا في هذه الرسالة...”

- Istanbul, Sulaymaniyah Library of Manuscripts, Yusufiyah, N° 2732 (Rosenfeld & Ihsanoğlu 2003: 207).

- Najaf, Iraq, the copy is very old (al-Ṭahrānī 1983: 249).

- Iraq, Library of the Iraqi Museum in two copies, one of which ends in Chapter VIII, No. 5/20866, and the other includes chapter VII, No. 2/23045, entitled *kāfiyah fī al-ḥisāb*, (al-Naqshabandī and ‘Abas 1980: 118-119).

- Mosul, Yaḥyā Pasha School, N° 237, entitled *Risālah kāfiyah fī al-ḥisāb* (al-Jalabī al-Muṣilī 1927: 244-245).

It seems that *Risālah kāfiyah* is an abridgment of al-Zanjānī’s book entitled ‘*Uddat al-Rā’ī*’.

The treatise contains ten chapters, it is not as detailed as ‘*Umdat al-Ḥussāb*’, it does not contain the theory of numbers, nor does it contain irrational numbers. We note that it has

several copies in the libraries of the world, which indicates the spread of this treatise.

### 3. *‘Uddat al-Rā’iq*

Al-Zanjānī talked about it in his book *‘Umdat al-Ḥussāb* (Al-Zanjānī, *‘Umdat al-Ḥussāb*, p. 60 r. (Topkapi, N° 3457)), and in his Treatise *Risāla Kāfiya fī ‘ilm al-Ḥisāb* (al-Zanjānī Msdb: 19 v.; al-Zanjānī, *Risāla Kāfiya fī ‘ilm al-ḥisāb*, p. 19 v. (Topkapi, N° 3152)), and in his book *Qusṭas al-mu‘ādalāh fī ‘ilm al-jabr wa-al-muqābalah* (Shahwī & Baqirī 2016: (پازده) (Shahwī & Baqirī, *Du Resala iz ‘Izz al-Dīn al-Zanjānī, ‘Umdat al-Ḥisab wa Qusṭas al-Mu‘adallah fī ‘Ilm al-Jabr wa Muqābala* (Two treatises by ‘Izz al-Dīn al-Zanjānī, *‘Umdat al-Ḥisab and Qusṭas al-Mu‘adallah fī ‘Ilm al-Jabr wa Muqābala*), 1395 AH/2016 AD), there is no information about it.

### 4. *Al-Risālah al-‘Izziyyah*

It is beginning by:

”باسم الله الرحمن الرحيم وصلى الله على محمد وآله وصحبه أجمعين، الحمد لله رب العالمين، وأما بعد فهذه رسالة في قواعد الحساب، حررتها على سبيل الاختصار ورتبتها على فصول وسميتها بالرسالة العزمية...”

In the copy of al-Ḥāhiriyyah Library (Damascus), it is ending by:

”هذه الرسالة لا تحتل البسط، فمن أراد الزيادة في كل باب فليرجع إلى بحر الفوائد في علم الحساب والقواعد والله تعالى أعلم“.

In the copy of Berlin Library, it is ending by:

”هذه الرسالة لا تحتل البسط، فمن أراد الزيادة في كل باب فليرجع إلى بحر الفوائد في علم الحساب والقواعد والحمد لله تعالى، واعلم أن كل مقدار ضرب في نفسه..... وعشرين شيئا وعشرين أحادا وانتهى بعون الله تعالى“.

There are copies of this treatise in:

- Berlin, Staatsbibliothek, Landberg 199, entitled *al-Risālah al-‘Izziyyah fī al-ḥisāb al-hawā’ī* (Catalog of Berlin Library vol. 5: 350).

- Damascus, al-Ḥāhiriyyah Library, in three copies N° 7759, N° 8358, and N° 6000 (The last copy is founded in the same collection with treatise *Risāla Kāfiya fī ‘ilm al-ḥisāb* occupied pages (98-114)), it is entitled *al-Risālah al-‘Izziyyah*, it is incomplete papers, and the date of the first copies is on 13 Sha’ban 955 AH in 18 folios, the second in the date of 1150 AH., numbering 7 papers, the third copied in 799 AH, their number 17 folios (‘Āyḍī 1973:10-12).

- Cairo, Dār al-kutub, Dār al-kutub majāmī’, N° 713, entitled *al-Risālah al-‘Izziyyah fī al-ḥisāb al-hawā’ī* (Rosenfeld & Ihsanoğlu 2003: 207).

The treatise contains 12 chapters, it have the same topics of *al- kāfiyah*. Perhaps he called it *al- ‘Izziyyah* like his book *al-Taṣrīf al-‘Izzī* in literature.

### 5. *Baḥr al-fawā’id fī ‘ilm al-ḥisāb wa al-qaw’id*

al-Zanjānī mentioned to it in his Treatise *al-Risālah al-‘Izziyyah* (al-Zanjānī Ms.: pp. 106 v., 108v. ; ‘Āyḍī 1973:10-12).

It seems that *Baḥr al-fawā’id fī ‘ilm al-ḥisāb wa al-qaw’id* is more detailed than *al-‘Izziyyah*, and this is what is indicated by al-Zanjānī at the end, where he said: “Whoever wants to detail every chapter, let him refer to *Baḥr al-fawā’id fī ‘ilm al-ḥisāb wa al-qaw’id*.”

### b. Magic Squares

#### *Risālah fī A’dād al-Wafq*

It was referred to as one of al- Zanjānī's works by: (Qara Bollut & Qara Bollut 2001 vol. 1: 37; Brockelmann 1977: 184; Rosenfeld & Ihsanoğlu 2003: 207), and it was in six chapters.

It is beginning by:

”... الحمد لله ... وبعد : فلما كان أعداد الوفق قد اشتهر ... طلب مَنِّي بعض أصحابي ... جمعت فيها ضوابط يعرف بها وضع أوفاق جميع الأعداد...”

It is ending by  
 ”فإن الذكي إذا أتقن ما ذكرناه لم يشكل عليه، والله أعلم، تمت الرسالة والحمد لله رب العالمين والصلاة والسلام على سيدنا محمد وآله أجمعين“.

Copies of the manuscript are in the following libraries:.

- Istanbul, Istanbul Köprülü Library, NONE (Köprülü), N° 828/2.
- Istanbul, Istanbul Millet Library, Feyzullah, N° 1362.
- Damascus, al- Zahirīyah Library, N° 11494, entitled *Risāla fī ‘ilm al-ḥisāb wa al-Falak*, this treatise is in 22 folios from a collection of 233 folios.
- USA, Arabic Manuscripts at Princeton University entitled *Risālah fī a‘dād ḍawābiṭ waḍ‘e al-wafq*, N° 5207, Conservation N° 976 (Mach 2011: 32).

In following places, this treatise entitled by *Risālat al-wafq al-tām* (Shahwī & Baqirī 2016: دوازه) (Shahwī & Baqirī, Du Resala iz ‘Izz al-Dīn al-Zanjānī, ‘Umdat al-Ḥisab wa Qustās al-Mu‘adallah fī ‘Ilm al-Jabr wa Muqābala (Two treatises by ‘Izz al-Dīn al-Zanjānī, ‘Umdat al-Ḥisab and Qustās al-Mu‘adallah fī ‘Ilm al-Jabr wa Muqābala), 1395 AH/2016 AD):

- Iran, Malak National Library, N° 37/1181.
- Iran, National Library of Iran, N° 11/2700.
- Copy in Tehran was copied in Dhul-Qi‘dah 1287 AH, the author's name is not mentioned in the preface.

‘Imād al-Dīn Yahya Kāshānī<sup>20</sup> translated this treatise into Persian, entitled *al-a‘dād wa al-awfāq*, and it consists of an introduction, two chapters, and a conclusion, it is in the library of Shaykh al-Islām Zanjānī (Shahwī & Baqirī 2016: دوازه) (Shahwī & Baqirī, Du Resala iz ‘Izz al-Dīn al-Zanjānī, ‘Umdat al-Ḥisab wa Qustās al-Mu‘adallah fī ‘Ilm al-Jabr wa Muqābala (Two treatises by ‘Izz al-Dīn al-Zanjānī, ‘Umdat al-Ḥisab and Qustās al-Mu‘adallah fī ‘Ilm al-Jabr wa Muqābala), 1395 AH/2016 AD).

This short treatise arranged into six sections. It contains tables and methods to explain the placement of *al-Wafq*.

### c. In Algebra

#### 1. *Qusṭās al-Mu‘adallah fī ‘Ilm al-Jabr wa-al-Muqābala*

According to the copy of Dublin, Ireland, Chester Beatty Library, and the copy of university of Utah (is not an original copy, perhaps in turn reproduced from the original manuscript)<sup>21</sup>, this book is beginning by:

”الحمد لله الذي أحاط بكل شئ علمه، أما بعد فإن العلوم الرياضية ترتقى إلى أصناف كثيرة وأنواع نافعة شريفة لكن أنفعها علوم الحساب..“

It is ending  
 ”أعنى من مائه وستة وثلاثون ألفاً وستمئة جزء من ذلك المخرج أعنى من مائة وثلاثة وستين ألف الف واحد وعشرين ألفاً وثمانمائة والله أعلم بالصواب“

This book has been written in the year 643 AH/ 1245 AD (al-Zanjānī Ms<sub>b</sub>: 299 r.). Copies of the manuscript are in the following libraries:

- Istanbul, Topkapı Sarayı Müzesi, Ahmet III, N° 3457 (Yadegari 1980: 403).
- Dublin, Ireland, Chester Beatty Library, N° 3927.

<sup>20</sup> Yahyā ibn Aḥmed al-Kāshī or al-Kashānī (was alive in 745 AH. / 1344 AD.) is a mathematician, writer. He died in Isfahān (Kaḥāla 1957 vol. 13: 184).

<sup>21</sup> The Copy of this book is in Institute of Arabic Manuscripts (Cairo), N° 1073, it is from the copy of Utah university, which is a copy from the original.

It contains following chapters: Chapter One on names and orders, chapter two on multiplication, chapter three on division, chapter four on addition and subtraction, chapter five on proportional numbers, chapter six on extracting roots of unknowns, chapter seven on demonstrated *Mu'āmarāt* (means identities), chapter eight on the six algebraic issues, chapter nine in algebraic issues returning to the introduction, chapter ten in Induction.

## **2. *Kitāb al-Borhān***

Al-Zanjānī mentioned to it in his book *Qusṭās al-mu'ādalāh fī 'ilm al-jabr wa-al-muqābalah* (Yadegari 1980: 403), there is no information about it, and it appears that is in subject of algebra.

### **d. In Geometry**

#### ***Kitāb al-Uṣūl fī al-Handasah***

Copies of the manuscript are in the following library:

- Baku, Azerbaijan, Academy of Sciences, Institute of MSS of Azerbaijan, two copies, B/ N° 2520, B/ N° 4280 (Rosenfeld & Ihsanoğlu 2003: 207).

e. In Astronomy

#### **1. *Muḥtaṣar fī Isti'māl al-Aṣṭurlāb***

- Leiden Academy, number 1091 (Brockelmann 1977: 184).
- London, SOAS Library, Special Collections, N°45806, entitled *al-Madkhal fī 'ilm al-aṣṭurlāb* (Rosenfeld & Ihsanoğlu 2003: 207).

#### **2. *Al-Tadhkirah al-Majdiyya***

Ibn al-Fawṭī mentioned to it as a book written by al-Zanjānī (Ibn al-Fuwaṭī 1996 vol. 1: 247), it seems that it is in science of *al-hay'ah*.

## **CONCLUSION**

'Izz al-Dīn al-Zanjānī lived and died during the first half of seventh century AH/ thirteenth century AD, in the spatial framework that is currently known as Central Asia and Iraq. The most prominent historical event at that time and place was undisputedly the Mongol invasion of the Islamic lands, which affected every aspect of life.

He is 'Izz al-Dīn 'Abd al-Wahhāb ibn 'mād al-Dīn Ibrāhīm ibn 'Abd al-Wahhāb ibn abū al-Ma'ālī al-Khazrajī al-Zanjānī. His scientific interests are grammar, prosody, eloquence, mathematics and astronomy, he was not a jurist, but this field was his father's specialization, as his father was a Shafī'i jurist. He moved between most famous places of sciences at that time. If we consider the historical context as we observe the places to which 'Izz al-Dīn al-Zanjānī traveled, we can suppose that his multiple travels were probably triggered for two purposes: fleeing from Mongols in order to finding stability in safe cities, and seeking knowledge, which was difficult in those circumstances.

In view of the period during which al-Zanjānī lived, it seems that he gradually coexisted with Mongol invasion. Through his writings and treatises on mathematics and literatures, we can clearly discern that he worked as a sheikh, first in Abbasid schools, perhaps in Mosul and later in Baghdad, and mostly he moved to Bukhara for this purpose, especially when Mongol rulers revived it and lavished on its schools. After Mongol invasion of Baghdad, it seems that he served under their command for his contact with Naṣīr al-Dīn al-Ṭūsī, the minister of Hulegu. It does not seem that al-Zanjānī interfered in matters of politics, he was preoccupied only by with sciences, learning and education.

Mosul is the most important city to which al-Zanjānī traveled and resided, it seems to be the city from which he took his knowledge, and he was influenced by its most famous

scholars, including ibn al-Khabbāz in literature. It seems that he was influenced by the encyclopedic scholar Kamāl al-Dīn Mūsā ibn Yūnis, especially in the field of mathematics and astronomy, which can be deduced through the topics and axes that his books dealt with in this field, in which Kamāl al-Dīn Mūsā ibn Yūnis was a scholar.

Al-Zanjānī's mathematical books varied between arithmetic (including calculus), algebra, geometry, and *A'dād al-Wafq*. His arithmetic (calculus) books and treatises were in types of *al-ḥisāb al-hawā'ī* and *al-ḥisāb maftūḥ* (There is not much difference between them), which are now known as mental calculus, and it is the type that is used especially in transactions. His writings seem to be directed to teachers and learners, and it may direct also to aṣḥāb al-dawāwīn.

Among al-Zanjānī's mathematical books, his arithmetic books were the most spread, which is evidenced by the large number of copies of his arithmetic manuscripts, with their extent in various Islamic regions and international libraries. Unfortunately, it seems that some of his other mathematical and astronomy books did not find the same spread, especially when compared to his literary works. Perhaps this was due to Mongol invasion of Baghdad, which overthrows the capital of Abbasid, the castle of sciences in the Arabic language; especially since al-Zanjānī moved to Tabriz after Mongol invasion, which is a Persian region in which scientists had started publishing in Persian.

As for his scientific and social standing, in view of his friendship with Naṣīr al-Dīn al-Ṭūsī, who fulfilled his desire to write a well-known book which had many commentaries as *al-Tadhkirah*, we can discern the high scientific and social status of al-Zanjānī, in addition to what was written by his biographers.

Al-Zanjānī's scientific level in mathematics is faultless, especially when we compare his arithmetic writings with writings of his contemporaries, as there are issues mentioned in his book *Umdat al-Ḥussāb* related to Diophantine equations (al-Zanjānī Ms<sub>a</sub>: 60 r.) that were difficult for his contemporaries such as ibn al-Khawām al-Baghdādī (643-736 AH./ 1246- 1336 AD.), who mentioned to some of these issues as intractable issues in his last book *al-Fawā'id al-Bahā'iyā* (Ibn al-Khawām al-Baghdādī Ms: 36 r.- 37 v), noting that al-Baghdādī's book spread and was a reference for the science of arithmetic in subsequent centuries.

'Izz al-Dīn al-Zanjānī is truly an outstanding person who affected and was affected by his scientific and social community, his scientific works need to shed more light and scientific editions.

## عز الدين الزنجاني وإسهاماته في الرياضيات والفلك

غرابية وسيلة\*

### ملخص

ستلقي هذه الدراسة الضوء على الحياة المجهولة لعالم رياضيات مَنِيِي، اكتسب شهرته من كتاباته وأعماله الأدبية التي سلطت الضوء على شخصيته العلمية، هو عالم الرياضيات عز الدين الزنجاني، الذي عاش في النصف الأول من القرن السابع الهجري/ الثالث عشر الميلادي في أثناء الغزو المغولي للشرق الإسلامي، وهي فترة متوترة ومظلمة من التاريخ الإسلامي في الشرق الإسلامي. ستحاول الدراسة أيضًا تعداد أعماله المختلفة في الرياضيات وعلم الفلك، مع ذكر مواقعها في مكتبات العالم المختلفة، والإشارة إلى بعض مؤلفاته التي نُسبت إلى غيره.

**الكلمات الدالة:** الزنجاني، نصير الدين الطوسي، ابن الفوطي، الغزو المغولي.

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## REFERENCES

- al-Amīn, Ḥasan (1997); *al-Isma'illiyūne wa-al-Maghūl wa Naṣīr al-Dīn al-Ṭusī*, Beirut: Markaz al Ghadīr lil-dirāsāt al-Islāmīyah, 2<sup>nd</sup> ed.
- Ahlwardt, W. (1893); *Catalogue of Berlin Library*, Vol. 5, Berlin: A. Asher and C<sup>o</sup>.
- al-'Abayjī, Maysoūn Ḍanoūn (May, 2009); "al-'Ālim al-Mawsū'īy Kamāl al-Dīn Mūsā ibn Yūnus ibn Mana'ah al-Moūṣilī (d.629 AH./1242 AD.: Dirāsah fī Sīratihī al-'Ilmīyah". *Majjalat Dirāsāt Mūṣilīyah*, vol. 24, Pp: 53- 88.
- 'Āyḍī, Muḥammad Ṣalāḥ (1973); *Fihris Makḥṭūṭāt Dār al-Kutub al-Zāhirīyah (Mathematics)*, Damascus: Majma' al-Lughah al-'Arabīyah.
- al-Bābānī, I. P. (1951); *Hadiyat al-'Ārifīne fī Asmā' al-Mu'alifīne*, Vol. 1, Istanbul: Wakalat al-Ma'ārif al-Jalīla.
- Brockelmann, K. (1977); *Tārīkh al-Adab al-'Arabī*, Vol. 5, 'Abd al-Ḥalīm al-Najjār Trans., Cairo: Dar al-Ma'ārif, 5<sup>th</sup> ed.
- al-Dhahabī, Shamas al-Dīn ibn Muḥammad (d. 748 AH./ 1348 AD.)(1919); *Tadhkirat al-Ḥuffāz*, Vol. 4, 'a.-R. al-Mo'alimī ed., Hyderabad: 'Ottoman Encyclopedia.
- al-Dhahabī, Shamas al-Dīn ibn Muḥammad (d. 748 AH./ 1348 AD.)(2013); *Tārīkh al-Islām wa-wafaiyāt mashāhīr al-A'lām*, Vol. 15, B. A. Ma'rūf ed., Beirut: Dār al-Gharb al-Islām.
- Fajjāl, Y. (1996); "al-Zanjānī : Ḥayātuhu wa Muṣannafātuhu". *Majallat 'Ālam al-Kutub*, vol. 1, no. 17, Pp: 34- 38.
- al-Hamadḥānī, Rashīd al-Dīn Faḍl Ullāh (d. 718 AH/ 1318 AD)(1960). *Jāmi' al-Tawārīkh*, Vol. 1, Muḥammad Hindāwī, Fo'ād al-Ṣayād and Muḥammad Nash'at Trans., Egypt: Dār Iḥyā' al-Turāth al-'Arabī.
- al-Ḥabashī, 'A. M. (2004); *Jami' al-Shurūḥ wa al-Hawāshi: Mu'jam shāmil li-Asmā' al-Kuub al-Mashrūḥah fī al-Turāth al-Islāmī wa-Bayān Shuruḥahā*, Vols. 1, 3, Abū Dhābī: al-Majma' al-Thaqāfi.
- Ḥājī khalīfah, M. (d. 1067 AH./1657 AD.)(N.D.); *Kashf al-Ḍunūn 'am Asāmī al-Kutub wa-la-Funūn (Suspicion Revealed)*X, Vols. 1, 2, M. C. Yaltafāyā & R. B. Al-Klīsī eds., Lebanon: Mu'asasat al-Tārīkh al-'Arabī.
- Ibn al-Athīr, Abū al-Ḥasan 'Alī ibn Abī al-karm Muḥammad (d. 630 AH./ 1233 AD.)(1987); *al-Kāmil fī al-Tārīkh*, Vol. 10, Beirut: Dār al-Kutub al-'Ilmīyah.
- Ibn 'Abd al-Kāfī, 'Ubāid (1913); *Sharḥ al-Maḥzun bi-hi 'alā ghayri 'Ahlihi (Doubtful explanation)*, A. A. al-Shanqītī & I. B. Yehuda eds., Egypt: al-Sa'adah Press, 1<sup>st</sup> ed.
- Ibn al-Fuwaṭī, Kamāl al-Dīn Abū al-Faḍl (d. 723 AH./ 1323 AD.)(1996); *Majma' al-Ādāb fī Mu'jam al-Asmā' 'alā Mu'jam al-Alqāb*, Vols. 1, 2, 4, M. al-Qāzīm ed., Tehran: Printing and Publication Institution, Ministry of Culture and Islamic Guidance.
- Ibn Khallikān, Shams al-Dīn Abū al-'Abās Aḥmad ibn Muḥammad ibn Ybrāhīm (d. 681 AH./ 1282 AD.)(1977); *Wafaiyāt al-A'yān wa Anbā' Abnā' al-Zamān*, Vols. 5, 6, Iḥsān 'Abbās ed., Beirut: Dār Ṣābir.
- Ibn al-Khawām al-Baghdādī, 'Imād al-Dīn (d. 736 AH./ 1336 AD.)(Ms); *al-Fawā'id al-Bahā'īyah*, Manuscript n°. We. 1129, occupies pages 1r.- 37v, Berlin Library.
- al-Jalabī al-Mūṣilī, Dāoūd (1972); *Makḥṭūṭat al-Mūṣil*, Iraq: Maṭba'at al-Furāt.
- Kaḥālāh, 'Umar Riḍah (1957); *Mu'jam al-Mu'alifīn*, Vols. 6, 13, Beirut: Maktabat al-Muthannā - Dār Iḥyā' al-Turāth al-'Arabī.
- Mach, R. (2011); *Catalogue of Arabic Manuscripts at Princeton University (Yahudaa Group)*, Vol. 8, M. 'Ayāsh ed. and M. 'Ayāsh Trans., Jeddah: Saqifat al-Ṣafā al-'Ilmīyah.
- Madelung, W. (2004); "al-Zanjānī". in *THE ENCYCLOPAEDIA OF ISLAM (NEW EDITION)*, XII, P. J. Bearman, T. Bianquis, C. E. Bosworth, E. V. Donzel, and W. P. Heinrichs eds., Pp: 841-842

- al-Naqshabandī, Usāmah Nāṣir and ‘Abas, Zamīā’ Muḥammad (1980); *Makḥṭūṭat al-Ḥisāb wa al-Handasah wa al-Jabr fī Maktabat al-Muthaf al-‘Irāqī*, Baghdād: Ministry of Culture and Information - General Organization for Antiquities and Heritage.
- al-Nuwayrī, Aḥmad ibn ‘Abd al-Wahāb ibn Muḥammad Shihāb al-Dīn (d. 733 AH./ 1333 AD.)(2004); *Niḥayāt al-Irb fī Funūn al-Adab*, Vols. 12, 26-27, Mufīd Qumīḥah & Others eds., Beirut: Dār al-Kutub al-‘Ilmīyah.
- Qara Bollut, ‘A. R., & Qara Bollut, A. T. (2001); *Mu‘jam Tārīkh al-Turāth al-Islāmī fī Maktabāt al-‘Ālam al-Makḥṭūṭat wa al-Maṭbū‘a*, Vols. 1, 3, Kayseri, Turkey: Dār al-‘Aqabah.
- Ridhawī, Mudaris Muḥammad Taqī (1998); *al-‘Allāmah al-Khawājah Naṣīr al-Dīn al-Ṭūsī: Hayātuhu wa Athāruhu*, ‘Ali Hāshim al-Asadī Trans., Mashhad: Printing and Publishing Institution of Astan Rizavi.
- Rosenfeld, B. A. & Ihsanoğlu, E. (2003); *Mathematicians, astronomers and other scholars of Islamic and their works (7th-19th c.)*, Istanbul: I.R.C.I.C.A.
- al-Subkī, Tag al-Dīn ‘Abd al-Wahāb (d. 756 AH./ 1355 AD.)(1964); *Ṭabaqāt al-Shāfi‘iyah al-Kubrāh*, Vol. 8, Maḥmoūd al-Ṭanaḥī & ‘Abd al-Fatāḥ al-Ḥelw eds., Egypt: Dār Fayṣal ‘Isā al-Bābī li-al-Nashr.
- Suter, H. (1900); *Die Mathematiker und Astronomen der Araber und ihre Werke*, Leipzig: Druck und verlag Von R, G, Teubner.
- al-Suyūṭī, Jalal al-Dīn ‘Abd al-Raḥmān (d. 911 AH./ 1505 AD.)(1979); *Bughyat al-Wu‘ah fī Ṭabaqāt al-Lughawīyn wa-al-Nuḥāh*, Vol. 1,2, Muḥammad abū al-Faḍl ed., Damascus: Dār al-Fikr, 2<sup>ed</sup> ed..
- Shahwī, P. K., & Baqirī, M. (2016); *Du Resala iz ‘Izz al-Dīn al-Zanjānī, ‘Umdat al-Ḥisab wa Qustās al-Mu‘adallah fī ‘Ilm al-Jabr wa Muqābala (Two treatises by ‘Izz al-Dīn al-Zanjānī, ‘Umdat al-Ḥisab and Qustās al-Mu‘adallah fī ‘Ilm al-Jabr wa Muqābala)*, M. Zamani Introd., Markaz Mīrāth Maktūb.
- al-Ṣallābī, ‘Alī Muḥammad (2009); *al-Maghūl bayna al-Intishār wa-al-Inkisār*, Vol. 1, Beirut: Dār al-Ma‘rifah.
- al-Ṭahrānī, Āqa Bazrak (1983); *al-Dhari‘ah ilā Taṣānīf al-Shi‘ah*, Beirut: Dār al-Aḍwā’, 3<sup>rd</sup> ed.
- al-Ṭūsī, Naṣīr al-Dīn Muḥammad ibn Muḥammad (d. 672 AH/ 1274 AD)(after 1256); *al-Tathkirah fī al-Hai‘ah*, Manuscript of Qatar National Library, Original British Library: Oriental Manuscripts, N° 11209.
- Yadegari, Mohammad (1980); “A Widespread Concept in Medieval Islamic Mathematics”. The Binomial Theorem: *Historia Mathematica*, vol.7, Pp: 401-406.
- al-Zanjānī, ‘Izz al-Dīn Abū al-Faḍā’il ‘bd al-Wahāb ibn Ibrāhīm al-Khazrajī (d. 660 AH./ 1262 AD.)(1991); Mi‘yār al-Nuzzār fī ‘Ulūm al-Ash‘ār (The Criterion of Observant in the Sciences of Poetry) (Vol. 1), Muḥammad ‘Alī Rizq al-Khafājī ed., Cairo: Dār al-Ma‘ārif.
- al-Zanjānī, ‘Izz al-Dīn Abū al-Faḍā’il ‘bd al-Wahāb ibn Ibrāhīm al-Khazrajī (d. 660 AH./ 1262 AD.)(Ms<sub>a</sub>); ‘Umdat al-Ḥisāb (Reliably of Arithemeticians), facsimile edition of manuscript no. 3457, Istanbul: Topkapı Sarayı Müzesi-Ahmet III.
- al-Zanjānī, ‘Izz al-Dīn Abū al-Faḍā’il ‘bd al-Wahāb ibn Ibrāhīm al-Khazrajī ((d. 660 AH./ 1262 AD.)(Ms<sub>b</sub>); Qustās al-Mu‘adallah fī ‘ilm al-Jabr wa-al-Muqābala, facsimile edition of manuscript no. 3927/4, Tabriz: MS Chester Beatty.
- al-Zanjānī, ‘Izz al-Dīn Abū al-Faḍā’il ‘Abd al-Wahhāb ibn Ibrāhīm al-Khazrajī (d. 660 AH./ 1262 AD.)(Ms<sub>c</sub>); al-Risālah al-‘Izīyah fī al-Ḥisāb al-Hawā’ī, Berlin: Staatsbibliothek Landberg 199.
- al-Zanjānī, ‘Izz al-Dīn Abū al-Faḍā’il ‘Abd al-Wahhāb ibn Ibrāhīm al-Khazrajī (d. 660 AH./ 1262 AD.)(Ms<sub>d</sub>); *Risālah Kāfiya fī ‘ilm al-ḥisāb al-hawā’ī*, Berlin: Staatsbibliothekx, Ms. or. Quart 100.
- al-Zarkalī, Khair al-Dīn ibn Maḥmoūd ibn muḥammad (2002); *al-A‘lām*, Vol. 4, Beirut: Dār al-‘Ilm lil-Malālīyn, 5<sup>th</sup> ed.